

In defense of local textures (and other Higgs gradients)

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(Dated: October 18, 2008)

Cruz et al. recently showed that the CMB cold spot can be explained by a GUT-scale texture. But following Turok's argument that gauged configurations always relax quickly, they posit a global symmetry, without obvious relation to GUTs. An observation by Nambu invalidates Turok's argument when the broken symmetry group has commuting generators. This is demonstrated explicitly in the standard model of electroweak interactions and holds generally for intermediate SSB stages in GUTs. The cold spot could therefore be due to a GUT texture, and electroweak Higgs gradients may evolve indefinitely.

PACS numbers: 11.27.+d, 98.80.-k, 12.10.-g, 11.15.Ex

INTRODUCTION

Are all gauge interactions unified at some high energy scale? Running the renormalization group equations of the Minimal Supersymmetric Standard Model (MSSM) under the assumption that supersymmetry shows up around 1 TeV , the couplings converge at $1.2 \times 10^{16}\text{ GeV}$ [1], suggesting the existence of a Grand Unified Theory (GUT) which undergoes spontaneous symmetry breaking (SSB) at that scale – alas, far beyond reach of accelerator experiments. It is therefore intriguing that the anomalous cold spot found by WMAP in the cosmic microwave background (CMB) can be explained by the collapse of a texture (a localized, knot-like field configuration) originating at a symmetry breaking scale $8.7 \times 10^{15}\text{ GeV}$ [2].

A window on otherwise inaccessible GUT physics may have opened, but there is a problem. GUTs are gauge field theories. The texture in [2], on the other hand, is produced by breaking a global symmetry (“global texture”).

The reason is the following argument, due to Turok [3]: a texture consists of a scalar multiplet $\Phi(x)$ taking values on a vacuum manifold \mathcal{M} (the bottom of some symmetric potential $V(\Phi)$), initially chosen randomly upon SSB in causally disconnected regions [4]. It can therefore be written $\Phi(x) = U(x)\Phi_0$, with Φ_0 an arbitrarily chosen constant on \mathcal{M} and U a local symmetry transformation. If Φ is gauged, i.e. if it is a Higgs field [5][6], the covariant derivative acting on it is $D_\mu(x) = \partial_\mu + ig\mathbf{W}_\mu(x)$, with \mathbf{W}_μ denoting the gauge fields and g the coupling constant. \mathbf{W}_μ can now “fall” to $\mathbf{W}_\mu = (i/g)(\partial_\mu U)U^{-1}$ at every point in space, making $D_\mu\Phi = 0$. The gradient energy of Φ is then zero. \mathbf{W}_μ is pure gauge, so its energy also vanishes and the configuration stops evolving.

Naively, this relaxation process should play out on the time scale typical of the gauge interaction, i.e. in a microphysical time, whereas the texture in [2] created the CMB cold spot by collapsing at redshift $z \sim 6$, a billion years after the big bang. So at first sight it can only be a global texture, and it is not at all clear how it might fit in a GUT scheme.

TWO CAVEATS

A first caveat to the naive guesstimate above was pointed out in [7]: even topologically trivial Φ configurations constrained to \mathcal{M} carry conserved quantities (energy, momentum, gauge currents, all in derivative terms) which must go elsewhere, i.e. to fermions, upon relaxation. Fermions can only be produced effectively while the energy and charges within the Compton volume of a fermion pair (e.g. electron + neutrino for electroweak interactions) are \geq the total mass and charges of such a pair (on shell). Once Φ gradients fall below this threshold, dissipation to fermions becomes exponentially suppressed. The evolution of Φ and \mathbf{W}_μ then becomes a Hamiltonian flow, but need not stop.

A second, subtler caveat follows from Nambu's “generalized Meissner effect” [8]: the condition $D_\mu\Phi = 0$ does not guarantee vanishing energy if a symmetry generator has a zero eigenvalue in the representation of Φ , i.e. if a subset \mathbf{A}_μ of \mathbf{W}_μ remains massless after SSB. To see this, apply the transformation law $\mathbf{W}_\mu \rightarrow \mathbf{W}'_\mu = U\mathbf{W}_\mu U^{-1} + (i/g)(\partial_\mu U)U^{-1}$ to a constant $\mathbf{W}_\mu = \mathbf{A}_\mu$. Rather than the unique \mathbf{W}_μ vacuum implicitly assumed by the naive argument, there is now a manifold of degenerate vacua $\mathbf{W}_\mu = U\mathbf{A}_\mu U^{-1} + (i/g)(\partial_\mu U)U^{-1}$. If each point in space picks \mathbf{W}_μ independently, there will be (generalized) electric and magnetic fields with finite energy density. Relaxation to vacuum can therefore not proceed independently at each point: the relaxation rate is limited by causality. This happens when the group is not simple or the representation's rank is ≥ 2 , i.e. for any realistic gauge field theory, since linear combinations of commuting generators then exist which annihilate Φ .

NON-LINEAR SIGMA MODEL

The global texture in [2] is modeled by an $O(4)$ non-linear sigma model (NLSM), the classical theory of a real 4-component Φ constrained to take values on a 3-sphere. The classical approximation can be motivated by noting

that in the low energy/long wavelength limit, the quantum effective action is dominated by stationary points of the action; SSB provides a simple example of background field quantization, with quantized short wavelength perturbations, i.e. particles, propagating over a semiclassical background of long wavelength modes [12][13]. The NLSM approximation holds because excitations in the non-flat direction of $V(\Phi)$ are suppressed by powers of interaction energy E_I over symmetry breaking scale E_{SB} . In practice the Lagrangian is split in two: a low energy semiclassical part for long wavelength modes with large occupation number (implying mass $\ll E_I$) and a high energy UV completion for particles.

All this remains true when Φ is a Higgs field. For $E_I \ll$ Higgs mass, a Higgsed gauge theory reduces to a gauged NLSM (GNLSM), which can be viewed as the leading, model-independent term of an effective Lagrangian with model-dependent higher order terms in E_I/E_{SB} [14]. Since it does not include radial excitations, this GNLSM remains valid (and classically exact) even if the Higgs field is only effective and no Higgs particle exists, as in technicolor models. In the electroweak example, since the minimal model is experimentally confirmed up to $E_I \sim 100 \text{ GeV}$ [15], whatever actually causes electroweak symmetry breaking must reduce to the same GNLSM for $E_I \ll 100 \text{ GeV}$, or equivalently for distances $\gg 10^{-18} \text{ m}$, with any corrections from unknown sectors suppressed by powers of $\sim E_I/(100 \text{ GeV})$ [16][17][18].

The electroweak GNLSM is conventionally written in “polar” field coordinates θ_a , so that

$$\mathbf{U} = \exp(i\theta_a \tau_a/2) = \cos(\theta/2) + i \frac{\theta_a \tau_a}{\theta} \sin(\theta/2) \quad (1)$$

with τ_a = Pauli matrices, $\theta = \sqrt{(\theta_1)^2 + (\theta_2)^2 + (\theta_3)^2}$ and

$$\begin{bmatrix} \phi^{0\dagger} & \phi^+ \\ -\phi^{+\dagger} & \phi^0 \end{bmatrix} = \frac{\nu}{\sqrt{2}} \mathbf{U} \quad (2)$$

where $\nu \simeq 246.3 \text{ GeV}$ is the symmetry breaking parameter. The covariant derivative acting on \mathbf{U} is

$$\mathbf{D}_\mu = \partial_\mu + i \frac{g_W}{2} W_\mu^a \tau_a - i \frac{g_B}{2} B_\mu \tau^3 \quad (3)$$

and the full Lagrangian is

$$\begin{aligned} \mathcal{L}_{EW} = & -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} \\ & + \frac{\nu^2}{4} \text{Tr} \left[(\mathbf{D}_\mu \mathbf{U})^\dagger (\mathbf{D}^\mu \mathbf{U}) \right] \end{aligned} \quad (4)$$

\mathcal{L}_{EW} has been used as is to compute the one-loop thermal effective action for an electroweak plasma at E_I between Higgs and weak gauge boson masses m_W and m_Z [19], but at low z we are interested in the low energy limit $E_I \ll m_W < m_Z$, where the massive gauge bosons can not be excited either and the semiclassical background

consists of massless modes only. Remembering Nambu’s lesson, we therefore look for a linear combination of gauge fields with zero effective mass, i.e. a generalized photon.

In the basis $[B_\mu, W_\mu^1, W_\mu^2, W_\mu^3]$, the \mathcal{L}_{EW} terms quadratic in B_μ and W_μ^a give rise to the mass matrix

$$\frac{\nu^2}{2} \begin{bmatrix} g_B^2 & g_B g_W \Theta_1 & g_B g_W \Theta_2 & -g_B g_W \Theta_3 \\ g_B g_W \Theta_1 & g_W^2 & 0 & 0 \\ g_B g_W \Theta_2 & 0 & g_W^2 & 0 \\ -g_B g_W \Theta_3 & 0 & 0 & g_W^2 \end{bmatrix} \quad (5)$$

where we have introduced the convenient auxiliary quantities

$$\Theta_1 = [\theta_1 \theta_3 (\cos(\theta) - 1) + \theta \theta_2 \sin(\theta)] / \theta^2 \quad (6)$$

$$\Theta_2 = [\theta_2 \theta_3 (\cos(\theta) - 1) - \theta \theta_1 \sin(\theta)] / \theta^2 \quad (7)$$

$$\Theta_3 = [(\theta_1^2 + \theta_2^2) \cos(\theta) + \theta_3^2] / \theta^2 \quad (8)$$

satisfying $(\Theta_1)^2 + (\Theta_2)^2 + (\Theta_3)^2 = 1$. The eigenvalues of Eq. (5) are the tree level masses squared of photon, W^\pm and Z^0 . The two degenerate eigenstates can be orthogonalized to obtain

$$A_\mu \propto \left[\frac{g_W}{g_B \Theta_3}, -\frac{\Theta_1}{\Theta_3}, -\frac{\Theta_2}{\Theta_3}, 1 \right] \quad (9)$$

$$\hat{W}_\mu^1 \propto \left[0, -\frac{\Theta_2}{\Theta_1}, 1, 0 \right] \quad (10)$$

$$\hat{W}_\mu^2 \propto \left[0, \frac{\Theta_1 \Theta_3}{\Theta_1^2 + \Theta_2^2}, \frac{\Theta_2 \Theta_3}{\Theta_1^2 + \Theta_2^2}, 1 \right] \quad (11)$$

$$Z_\mu \propto \left[-\frac{g_B}{g_W \Theta_3}, -\frac{\Theta_1}{\Theta_3}, -\frac{\Theta_2}{\Theta_3}, 1 \right] \quad (12)$$

Inverting Eqs. (9)-(12) and setting $\hat{W}_\mu^1 = \hat{W}_\mu^2 = Z_\mu = 0$ to account for the decay of all massive modes yields

$$B_\mu = A_\mu \cos(\theta_W) \quad (13)$$

$$W_\mu^1 = -A_\mu \Theta_1 \sin(\theta_W) \quad (14)$$

$$W_\mu^2 = -A_\mu \Theta_2 \sin(\theta_W) \quad (15)$$

$$W_\mu^3 = A_\mu \Theta_3 \sin(\theta_W) \quad (16)$$

where θ_W is the Weinberg angle, $\sin(\theta_W) = g_B / \sqrt{g_B^2 + g_W^2} \simeq \sqrt{0.2216}$. Substituting Eqs. (13)-(16) into \mathcal{L}_{EW} then yields our final, low energy effective Lagrangian for the electroweak boson sector

$$\begin{aligned} \mathcal{L} = & \frac{\nu^2}{8} \left[\partial_\mu \theta \partial^\mu \theta + \frac{4 \sin^2(\theta/2)}{\theta^2} \left(\vec{\theta} \times \partial_\mu \vec{\theta} \right)^2 \right] \\ & - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ & - \frac{\sin^2(\theta_W)}{4} (A_\mu \partial_\nu \Theta_a - A_\nu \partial_\mu \Theta_a)^2 \end{aligned} \quad (17)$$

with $\vec{\theta} = [\theta_1, \theta_2, \theta_3]$, $\theta = |\vec{\theta}|$.

The first row in Eq. (17) is just the plain $O(4)$ NLSM in polar field coordinates, the second row is the Maxwell

Lagrangian, the third row couples them $\propto \sin^2(\theta_W)$, acting as an effective photon mass term when $\vec{\theta}$ is not constant. Analogous Lagrangians (with larger symmetries and more fields) can be expected to describe the long wavelength modes of GUTs at intermediate symmetry breaking stages.

GAUGE FIXING

Eq. (17) does not include gauge fixing terms; a physical gauge, i.e. a gauge which does not introduce fictitious fields, is therefore implied. Most convenient (and popular) is the time-axial gauge $A_0 = 0$ [20][21][22][23][24][25] (note that in quantum theory, the unitary gauge is the singular limit of 't Hooft's R_ξ gauges, which require the introduction of ghosts). The electric and magnetic fields are then $\vec{E} = -\partial_0 \vec{A}$, $\vec{B} = \nabla \times \vec{A}$, and the energy density is a manifestly non-negative sum of quadratic forms,

$$\begin{aligned} \rho = & \frac{1}{2} (\vec{E}^2 + \vec{B}^2) \\ & + \frac{1}{2} \partial_0 \vec{\theta}^T (\nu^2 \mathbf{G} + \vec{A}^2 \mathbf{H}) \partial_0 \vec{\theta} \\ & + \frac{\nu^2}{2} \partial_m \vec{\theta}^T \mathbf{G} \partial_m \vec{\theta} \\ & + \frac{1}{2} (\varepsilon_{jkl} A_k \partial_l \vec{\theta})^T \mathbf{H} (\varepsilon_{jmn} A_m \partial_n \vec{\theta}) \end{aligned} \quad (18)$$

where \mathbf{G} , with components

$$G_{ab} = \left(\frac{\delta_{ad} \delta_{be}}{2} + \frac{1 - \cos \theta}{\theta^2} \varepsilon_{cda} \varepsilon_{ceb} \right) \frac{\theta_d \theta_e}{\theta^2} \quad (19)$$

is the 3-sphere metric, with eigenvalues $1/4$ and (doubly degenerate) $0 \leq (1 - \cos(\theta))/(2\theta^2) \leq 1/4$, while \mathbf{H} , with components

$$H_{ab} = \sin^2(\theta_W) \frac{\partial \Theta_c}{\partial \theta_a} \frac{\partial \Theta_c}{\partial \theta_b} \quad (20)$$

also has no negative eigenvalues (but zeros along all axes of $\vec{\theta}$ space). By the spectral theorem, $\vec{A} \neq 0$ can therefore only increase ρ for a given $\vec{\theta}$.

Varying \mathcal{L} in A_0 yields the Gauss constraint

$$\partial_0 \partial_m A_m = \partial_0 \vec{\theta}^T \mathbf{H} A_m \partial_m \vec{\theta} \quad (21)$$

(a plane in \vec{A} space) which completely fixes the gauge, leaving \vec{A} with only two independent degrees of freedom. As in any gauge other than unitary, the Goldstone modes remain. Keep in mind that we are working with non-perturbative, background *fields*; perturbations on this background (described by the UV completion not shown here) damp out within a Compton wavelength of the massive gauge bosons, so there is no plague of massless Goldstone *particles* propagating over macroscopic distances (see e.g. [26] for an explicit demonstration in the space-axial gauge). Those are confined, either to the background or to individual massive gauge bosons.

DISCUSSION

Since $\vec{A} \neq 0$ can only increase ρ for a given $\vec{\theta}$, it is tempting to set $\vec{A} = 0$ and coopt the wealth of existing work on the plain NLSM from hadron physics [27][28][29][30], cosmology [31], gravity [32][33][34][35] and general mathematical physics [36][37][38][39][40][41]. From this work, we know families of solutions on various metrics (and that any solution of the massless Klein-Gordon equation on the target spacetime [42] can be used to generate solutions along geodesics of \mathcal{M} [32]); that singularities never occur for suitably small initial data [33], making total decay to fermions unlikely (pair production makes large data small, but shuts down below a threshold); and that collapse can be prevented by rapid metric expansion [35]. From the point of view of [7], the existence of harmonic maps with polyhedral symmetry [30][43] is particularly interesting.

But we also know that electroweak texture collapse can be prevented by gauge interactions [44], and as Nambu taught us, there may be local minima with $\vec{A} \neq 0$ (e.g. cosmic magnetic fields coupled to $\vec{\theta}$ vortices). For GUTs, with more massless gauge fields at intermediate SSB stages, the space of possible solutions is also larger (but there may also be NLSM subgroups which do not couple directly to the photon).

Even the lowly standard model may have more surprises in store. To start with, the $\sin^2(\theta_W)$ term in Eq. (17) couples $\vec{\theta}$ to a heat bath of CMB photons. By inspection of the equations of motion, any NLSM solution satisfying $\partial_\mu \partial^\mu \theta_a = \partial_\mu \theta_a \partial^\mu \theta_b = 0$ (e.g. a plane wave) is also a (stochastic) GNLSM solution for constant $\langle A_m \rangle = 0$, $\langle A_m A_n \rangle = A^2 \delta_{mn}$, but what about the general case? Does radiation pressure significantly affect the dynamics, e.g. driving transition layers between regions of constant $\vec{\theta}$ (domain boundaries) to $\theta = 0, 2\pi$, where \vec{A} decouples?

When fermions are included, there is also a neutrino heat bath to consider, along with the real jokers: the quark condensates $\langle \bar{q}q \rangle$ which form at the chiral phase transition of QCD [45], with their own NLSM coupled to the GNLSM through \mathbf{D}_μ and Yukawa terms. Intriguingly, it is evident by inspection of the latter that the manifest \mathbb{Z}_2 symmetry of Eq. (17) under $\vec{\theta} \rightarrow -\vec{\theta}$ is broken by $\langle \bar{q}q \rangle \neq 0$. In the two-flavor case, with condensates $(\sigma, \pi_a \tau_a)$, it is also easily verified that $\theta = 0$ (by definition, our vacuum) can not be a minimum of the QCD-induced potential unless $\pi_a = 0$, leaving only $\sigma \neq 0$. Since the phases of quark mass terms and condensates are related to the strong CP-violation parameter θ_{QCD} through the axial anomaly [46][47], it is fair to wonder whether a solution of the long-standing strong CP problem may be lurking in this interplay between $\langle \bar{q}q \rangle$ and $\vec{\theta}$. Could the latter stand in for the stubbornly undetected and theoretically problematic [48] global axion [49]?

Finally, gravity may be important both locally, as a

stabilizer counteracting dispersion [50][51][52], and cosmologically. The pressure to ρ ratio of an isotropic “fluid” of scalar field configurations goes from $1/3$ for relativistic wave fronts to $-1/3$ in the static limit. A flat FRW cosmology dominated by such configurations could therefore evolve naturally from radiative (scale factor $a(t) \propto \sqrt{t}$) through “dust matter” ($a(t) \propto t^{2/3}$) to linear ($a(t) \propto t$). Incidentally, a linearly coasting cosmology [53] is known to fit observation well [54][55][56] while avoiding some problems of the concordance model [57][58]. A major objection might be that the photon conversion mechanism of [7] would then result in excessive SNe Ia dimming.

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I am grateful to Thomas Schaefer, who vehemently disagrees with this paper, for calling my attention to a boneheaded error in the first version.